Fresnel reflection and transmission at a planar boundary from media of equal refractive indices

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Some interesting properties of the Fresnel equations governing the reflection of plane electromagnetic waves at an interface between differing media are presented. The phenomenon of reflection that is independent of angle of incidence is shown to be theoretically possible if the media possess different magnetic permeabilities.

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Consider a plane interface separating two media of real indices of refraction n and n', where $n^2 = \epsilon\mu$ and $n'^2 = \epsilon'\mu'$, for ϵ,ϵ' being the dielectric constants of the media and μ,μ' the magnetic permeabilities. Consider a plane electromagnetic wave of the form $\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}$ incident onto the interface at an angle i from the unit normal pointing out of the interface into the incident medium. Here $\mathbf{k}$ is the propagation vector for the plane wave and ω is the angular frequency. The intent of this letter is to bring attention to an interesting special case of reflection and transmission of electromagnetic radiation from an interface.

The imposition of boundary conditions at the interface gives the familiar Snell law of refraction when all phase factors are equated, i.e., $n \sin(i) = n' \sin(r)$, where r denotes the angle of the transmitted wave.[1] Further, the imposition of continuity of the tangential components of the fields **E** and **H** yields the well-known Fresnel coefficients which relate the ratio of the reflected and transmitted field amplitudes to the incident field amplitudes. There are two possible polarizations: p polarization for the electric field vector parallel to the plane of incidence and s polarization for the electric field perpendicular to the plane of incidence. For each polarization the Fresnel coefficients, for reflected and transmitted amplitudes, are in general different.

The analysis of the Fresnel formulae is well documented in the literature; curves depicting the behavior of the reflected field amplitudes and phase change upon reflection for a large variety of cases are given in Ref. 2; curves for the reflectance, phases, and the behavior of the Brewster angle are presented in Ref. 3. (In this latter reference, the author defines three Brewster angles, and discusses how they behave for the case where the reflection medium possesses nonzero extinction.) However, most studies usually make the assumption that the magnetic permeabilities of the two media have a ratio of unity, i.e., $\mu/\mu' = 1$. This is a very good assumption throughout the visible spectrum, though may not be in the far infrared or millimeter domain.

An unusual situation occurs if we set the real indices of refraction equal, $n = n'$. Contrary to intuition, in the general form of the Fresnel coefficients there may still be nonzero reflectance from the interface as this equality is equivalent to the ratio

$$\mu/\mu' = \epsilon'/\epsilon. \tag{1}$$

Equality of the indices also says that the phase velocity of the electromagnetic radiation is the same in each medium since $v_p = c/n$ and $v_p' = c/n'$, and hence $v_p = v_p'$. Also, there is no refraction at the interface since from Snell's law $i = r$ (this also follows from the equality of phase velocities). However, the electromagnetic admittances of the two media, defined to be $y^2 = \epsilon/\mu$ and $y'^2 = \epsilon'/\mu'$, are not equal. Equality of the indices also gives the result (from the Fresnel coefficients) that the reflectance from the interface is a constant, independent of polarization, and independent of the angle of incidence. The reflection coefficient is given by

$$r = (y - y')/(y + y'), \quad i < \pi/2, \tag{2}$$

and the transmission coefficient is $t = 1 + r$ (in the absence of extinction in both media). This result is quite intriguing.

Though it appears that r is a constant for all angles of incidence, and hence t also, the analysis breaks down as we approach grazing incidence, i.e., as $i \to \pi/2$. We can see what happens by examining the general Fresnel equations where we let $n' = n + \delta$, for δ defined to be a small perturbation parameter that becomes vanishingly small. If we define r_s as the reflectivity for the s polarization and r_p as the reflectivity for the p polarization, then Fresnel's formulae take on the form

$$r_s = \frac{\mu'/\mu - (1 + 2\delta/n \cos^2 i + \delta^2/n^2 \cos^2 i)^{1/2}}{\mu'/\mu + (1 + 2\delta/n \cos^2 i + \delta^2/n^2 \cos^2 i)^{1/2}} \tag{3a}$$

and

$$r_p = \frac{\mu/\mu' - [n/(n+\delta)\cos i][1 - n^2 \sin^2 i/(n+\delta)^2]^{1/2}}{\mu/\mu' + [n/(n+\delta)\cos i][1 - n^2 \sin^2 i/(n+\delta)^2]^{1/2}}. \tag{3b}$$

In Fig. 1 are illustrated a variety of cases for the quantity $R_s = r_s r_s^*$, and in Fig. 2 we show several cases for $R_p = r_p r_p^*$. Note that for each polarization there may exist a Brewster angle, and if $\delta < 0$, the Brewster angle is followed by a jump to unity reflectance as the critical angle is passed for R_p.[4] Further, as $\delta \to 0$ we observe that the Brewster angle and subsequent jump to unity (either at the critical angle, or at grazing incidence) becomes more and more compressed toward $i = \pi/2$. When $\delta = 0$, the curves becomes singular at $i = \pi/2$ as the limit $\delta/\cos^2 i \to \infty$. It should be pointed out, however, that for a real physical situation it is unlikely that $\delta = 0$, but would possibly be very small.

Another interesting case that we should point out, which is well known,[5] is when the electromagnetic admittances of both media are equal, or equivalently

$$\mu'/\mu = \epsilon'/\epsilon. \tag{4}$$

Hence the reflectance is the same for both polarizations and is zero at normal incidence. As the angle of incidence increases the reflectance increases and goes to unity as grazing incidence is reached. If $\epsilon' < \epsilon$, the phenomenon of total internal reflection occurs when $i \geqslant \sin^{-1}(\epsilon'/\epsilon)$.

For all of the above situations, the magnetic permeabilities of the media must satisfy the condition that their ratio is

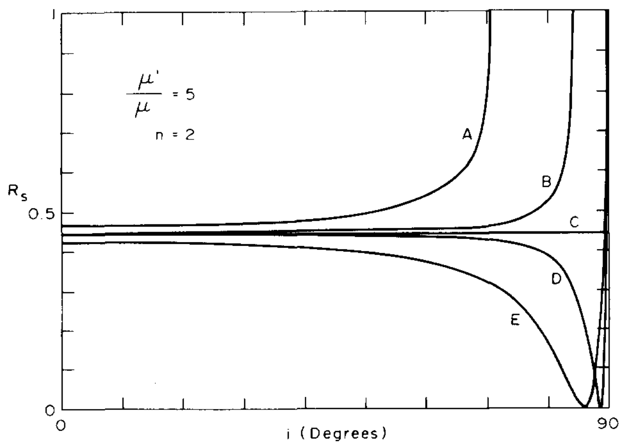

FIG. 1. Reflectance curves for s-polarized radiation. Here various values of δ are given: A: $\delta = -0.1$, B: $\delta = -0.01$, C: $\delta = 0.0$, D: $\delta = 0.01$, E: $\delta = 0.1$.

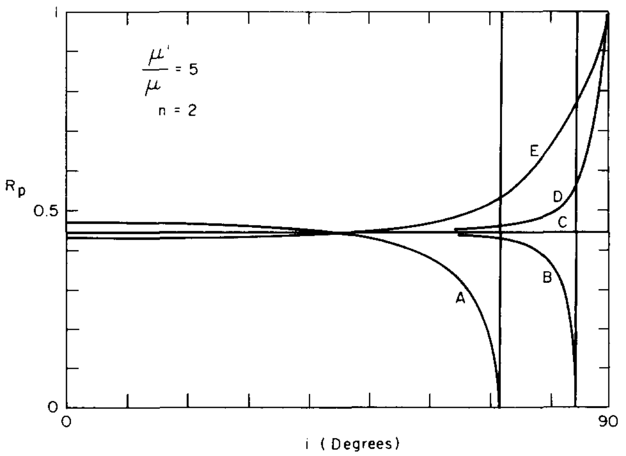

FIG. 2. Reflectance curves for p-polarized radiation. Here various values of δ are given: A: $\delta = -0.1$, B: $\delta = -0.01$, C: $\delta = 0.0$, D: $\delta = 0.01$, E: $\delta = 0.1$.

not unity, and such that relations (1) or (4) are satisfied. If, as is true with all known materials in the visible spectrum, $\mu' = \mu$, then conditions (1) or (4) cannot be satisfied, unless, of course, there is no interface. It is the author's desire that some of the observations in this letter will find some interest amongst millimeter wave and far infrared researchers. One interesting application may be in the design of mechanical or electrically driven optical switches. For example, from Fig. 2 we notice that the jump from the Brewster angle to critical angle occurs over an interval less than one degree. This, in effect, is analogous to an "on-off" state. Further, if the intrinsic media constants can be electrically tuned, it may be possible to rapidly switch from one curve to the other without recourse to mechanical means.

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