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BREWSTER ANGLES FOR MAGNETIC MEDIA

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Brewster angles (angles for which there is no reflection of an incident electromagnetic plane wave) will exist for either incident polarization at a planar interface separating two media provided that either or both of these media are magnetic. Except at certain angles (normal and grazing incidence) and specific material properties, a Brewster angle cannot exist simultaneously for both incident polarizations, but always exists for one polarization or the other for real values of permittivity and permeability. The existence conditions for the Brewster angle are readily formulated in terms of relative media permittivity and permeability, or relative media refractive index and electromagnetic impedance. When the ratio of permeabilities and permittivities is not unity, the Brewster angle is no longer orthogonal to the transmitted beam angle. From the viewpoint of the dipole oscillator model, the contributions of the electric and magnetic dipoles are such that they do not independently dominate the Brewster angle existence; rather the ratio of media permittivities and/or permeabilities determine when a Brewster angle may exist.

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INTRODUCTION

Recently, interesting and unusual properties of the reflection and scattering of electromagnetic plane waves in magnetic materials have been reported. It has been shown that for a planar interface and the special case of equal refractive indices for both magnetic media, the amplitude reflection coefficient is constant, and independent of the angle of incidence and incident polarization[1]. For the scattering of a plane wave from a magnetic sphere, the scattered wave can be zero for various scattering angles for certain specific ratios of permittivity and permeability.[2] This paper describes another interesting property of nonabsorbing magnetic materials, the existence of a Brewster angle (incident angle of total transmission or zero reflection) for either incident polarization of an electromagnetic plane wave. This variation of the Brewster angle with polarization is not well-known.[3] This is because most studies of the Fresnel reflection equations begin with the assumption of unity permeability which is true for the visible and infrared regions of the spectrum. In the submillimeter to radio-frequency parts of the spectrum one cannot necessarily ignor the permeability; indeed, the permeability may be complex.[4,5,6] For the case where dielectrics are considered, most treatments which incorporate the permeability usually go no further than a simple statement of derived equations (such as for the Brewster angles) and there is no detailed discussion of the influence of the permeability or the physical interpretation.[7,8] We expect that some of the results discussed here can be verified experimentally with materials such as the Yttrium Iron Garnets.[6]

We describe the material conditions that determine which incident polarization has a Brewster angle and show that the Brewster angle cannot exist simultaneously for both incident polarizations, except at normal and grazing incidence. We also show that whether or not the Brewster angle exists for either polarization is determined by the relative strengths of the ratios of permittivity and permeability or ratios of refractive index and bulk impedance. Further, in general the angle between the Brewster angle and the transmitted wave deviates from 90° whenever the ratio of permittivities and permeabilities is different from unity.

ANALYSIS

Consider the reflection and transmission of an incident electromagnetic plane wave at a planar interface separating two media, either or both of which may be magnetic. The constitutive relation between the electric field E and its displacement field D is linear, isotropic, homogeneous and time-independent, i.e. $D = \varepsilon E$ where ε is the medium permittivity. By magnetic, we mean that the constitutive relation between the magnetic field H and its induction field B is linear, isotropic, homogeneous and time-independent, i.e. $B = \mu H$ where μ is the medium permeability. We assume that for either or both media the permeability μ is not equal to μ_0, the permeability of free space and that both ε and μ are real and positive. Note that materials which closely approximate such properties exist at specific frequencies in the far infrared, millimeter, and microwave spectral regions.

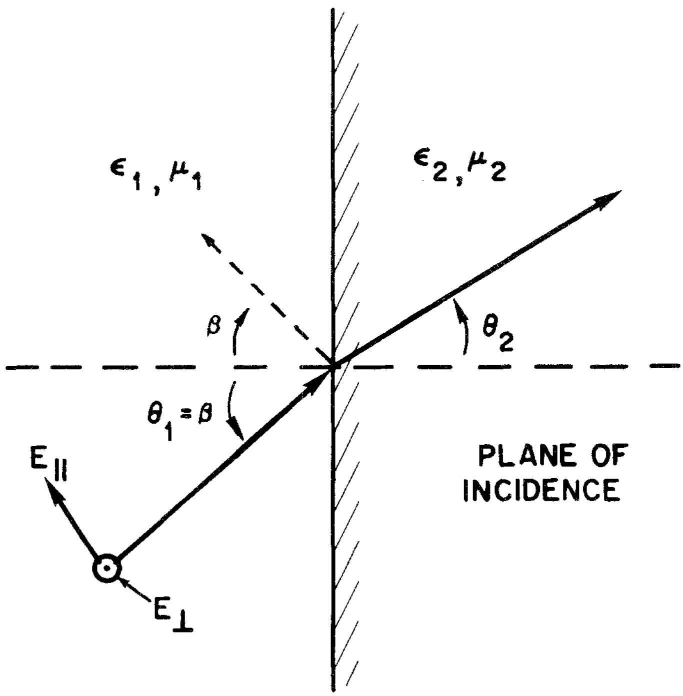

Fig. 1. Incident polarizations, media permittivities and permeabilities, angles of incidence and refraction, and Brewster angle are defined.

The reflection coefficients for either polarization are easily derived.[10,11] We let $r_\perp$ and $r_\parallel$ denote the electric field reflection coefficients for the polarizations respectively perpendicular and parallel to the plane of incidence. See figure 1 for further clarification. Define ε_1, μ_1 and ε_2, μ_2 as the permittivity, permeability pairs for the incident and reflecting media. Further analysis is simplified if we define dimensionless ratios for permittivity and permeability such that $\varepsilon = \varepsilon_2/\varepsilon_1$ and $\mu = \mu_2/\mu_1$, i.e. ε is the ratio of permittivity of the reflecting medium to the permittivity of the incident medium and similarly for μ. Now ε and μ are independent of the choice of units and can be interpreted as the relative strengths of the electric and magnetic dipole contributions of each media.[12] The reflection coefficients have the form[8]:

$$r_\perp = \frac{(\sqrt{\mu/\varepsilon}) \cos \theta_1 - \cos \theta_2}{(\sqrt{\mu/\varepsilon}) \cos \theta_1 + \cos \theta_2} \qquad (1)$$

$$r_\parallel = \frac{\cos \theta_1 - (\sqrt{\mu/\varepsilon}) \cos \theta_2}{\cos \theta_1 + (\sqrt{\mu/\varepsilon}) \cos \theta_2} \qquad (2)$$

where θ_1 and θ_2 are the angles of incidence and refraction. The value of the reflection angle θ_1 for which $r_\perp$ or $r_\parallel$ is zero is defined as the Brewster angle for that polarization (all polarizations thus denoted are for the electric field) and will be defined as $\beta_\perp$ or $\beta_\parallel$. Using Snell's law, $\sin \theta_1 = \sqrt{\varepsilon\mu} \sin \theta_2$, and setting equations (1) and (2) equal to zero, we find the following expressions for the Brewster angle:

$$\text{polarization } E_\perp : \quad \tan^2 \beta\perp = \mu \frac{(\varepsilon-\mu)}{1-\varepsilon\mu} \qquad (3)$$

$$\text{polarization } E_\parallel : \quad \tan^2 \beta_\parallel = \varepsilon \frac{(\mu-\varepsilon)}{1-\varepsilon\mu} \qquad (4)$$

Another form expresses the Brewster angles in terms of the relative refractive index m and relative bulk electromagnetic impedance z, where $m \equiv \sqrt{\varepsilon\mu}$ and $z \equiv \sqrt{\mu/\varepsilon}$. Equations (3) and (4) may now be written as

$$\text{polarization } E_{\perp} : \quad \tan^2 \beta_{\perp} = m^2 \frac{1-z^2}{1-m^2} \tag{5}$$

$$\text{polarization } E_{\parallel} : \quad \tan^2 \beta_{\parallel} = \frac{m^2}{z^2} \frac{z^2-1}{1-m^2} \tag{6}$$

From these equations, certain special cases for the Brewster angle are immediately obvious and reassuring. Equations (3) and (4) yield

$$\{\varepsilon \neq 1,\ \mu = 1\} \rightarrow \tan^2 \beta_{\parallel} = \varepsilon, \quad \beta_{\perp} \text{ does not exist,}$$

$$\{\varepsilon = 1,\ \mu \neq 1\} \rightarrow \tan^2 \beta_{\perp} = \mu, \quad \beta_{\parallel} \text{ does not exist.} \tag{7}$$

As is well-known, the Brewster angle for dielectrics ($\mu_1 = \mu_2 = \mu = 1$)exists only for the parallel polarization. However, the analogous magnetic case ($\varepsilon = 1$) has a Brewster angle only for the perpendicular polarization. Inspection of equations (5) and (6) show that for $z = 1$ (impedance matching), Brewster angles occur for both polarizations at normal incidence. For $m = 1$ (refractive index matching), Brewster angles occur for both polarizations at grazing incidence.[1] However, the existence of the latter case is difficult to show except in the limit of one refractive index approaching the other.

General conditions for the existence of the Brewster angle for either or both polarizations can be readily derived from equations (3) and (4) or equations (5) and (6) by requiring the right side of these equations to be positive and real. From equations (3) and (4), we have the following existence conditions:

$$\beta_{\perp} \text{ exists if } \mu \geqslant \varepsilon \geqslant 1/\mu \quad \text{or} \quad \mu \leqslant \varepsilon \leqslant 1/\mu,$$

$$\beta_{\parallel} \text{ exists if } \varepsilon \geqslant \mu \geqslant 1/\varepsilon \quad \text{or} \quad \varepsilon \leqslant \mu \leqslant 1/\varepsilon; \tag{8}$$

and from equations (5) and (6):

$$\beta_\perp \text{ exists if } \begin{bmatrix} m \geq 1 \\ z \geq 1 \end{bmatrix} \text{ or } \begin{bmatrix} m \leq 1 \\ z \leq 1 \end{bmatrix} ,$$
$$\beta_\parallel \text{ exists if } \begin{bmatrix} m \geq 1 \\ z \leq 1 \end{bmatrix} \text{ or } \begin{bmatrix} m \leq 1 \\ z \geq 1 \end{bmatrix} . \quad (9)$$

These inequalities are graphically portrayed as decision regions in figures 2 and 3. Equations (8) and (9) show that for real values of m and z or ε and μ, a Brewster angle always exists for one polarization. Note that many

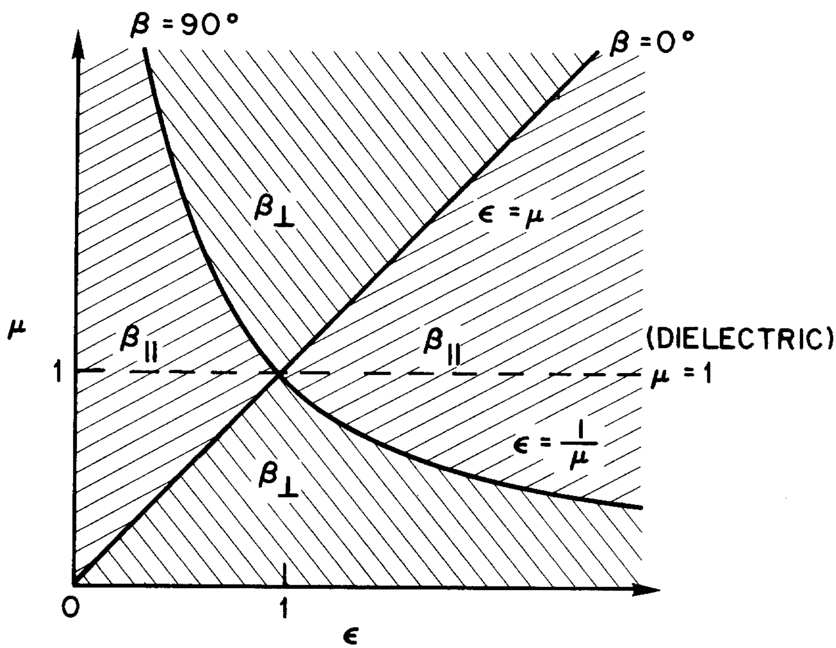

Fig. 2. Decision regions are illustrated for Brewster angles for different polarizations as determined by the relative permittivity ε and relative permeability μ. The regions denoted by $\beta_\perp$ define values of ε and μ where the Brewster angle exists for the electric field polarization perpendicular to the plane of incidence. Similar regions denoted by $\beta_\parallel$ are for the parallel polarization. The solid lines indicate where a Brewster angle exists for both polarizations. The dielectric case is denoted by the dotted line.

values of $(\varepsilon_1,\varepsilon_2)$ and (μ_1,μ_2) could represent a single point (ε,μ) in figure 2. A similar situation exists for (m,z) in figure 3. In each figure, the dotted line represents the dielectric $(\mu_1 = \mu_2 = \mu = 1)$ case. Actually $\mu = 1$ does not mean that both media have to be dielectrics. In fact, both media may be magnetic but their permeabilities must be equal. These materials are then magnetically equivalent but electrically dissimilar. A similar situation occurs for $\varepsilon = 1$. In figure 3, the point $m = z = 1$ (both media electromagnetically identical) has a Brewster angle

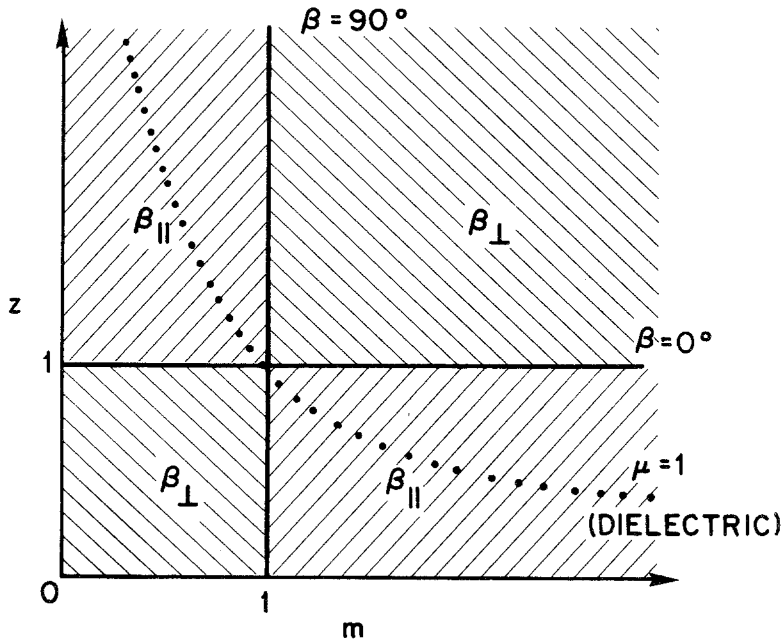

Fig. 3. Decision regions are illustrated for Brewster angles for different polarizations as determined by the relative refractive index m and relative impedance z. The regions denoted by $\beta_\perp$ define values of m and z where the Brewster angle exists for the electric field polarization perpendicular to the plane of incidence. Similar regions denoted by $\beta_\parallel$ are for the parallel polarization. The solid lines indicate where a Brewster angle exists for both polarizations. The dielectric case is denoted by the dotted line.

for all angles. A similar point exists at $\varepsilon = \mu = 1$ in figure 2.

For both polarizations the critical angle θ_c is defined by $\sin \theta_c = m = \sqrt{\varepsilon\mu}$ and exists if $m \leqslant 1$ or $\varepsilon\mu \leqslant 1$. From figures 2 and 3 we see that a critical angle can coexist with a Brewster angle providing the above conditions are satisfied. From the definition of the critical and Brewster angles, we have the relations:

$$\tan \beta_{\perp} = \sqrt{(1-\mu/\varepsilon)} \tan \theta_c$$

$$\tan \beta_{\parallel} = \sqrt{(1-\varepsilon/\mu)} \tan \theta_c .$$

Since the existance conditions for $\beta_{\perp}$ and $\beta_{\parallel}$ imply that respectively $\mu/\varepsilon < 1$ and $\varepsilon/\mu < 1$, the Brewster angle is always smaller than the critical angle.

DISCUSSION

With figures 2 and 3, we can interpret the material conditions which determine the existence of a Brewster angle for a particular polarization. Figure 3 implies that if both the refractive index and impedance of the reflecting medium are jointly greater than or less than the refractive index and impedance of the incident medium, the Brewster angle exists only for the perpendicular polarization. If, on the other hand, the relative refractive index and the relative impedance are <u>not</u> jointly greater than unity or less than unity, the Brewster angle exists only for the parallel polarization. For the special cases of $z = 1$ and $m = 1$, the Brewster angle is respectively at $\beta = 0^{\circ}$ and $\beta = 90^{\circ}$ for both polarizations.

If we can describe the material properties of ε and μ to be due primarily to the electric and magnetic dipole moments,[12] figure 2 offers some unusual interpretations of the existence conditions. The relative magnitudes of the electric or magnetic dipoles do not independently dominate the Brewster angle existence, contrary to what one might expect. The situation is more complicated. For example, one condition for the existence of $\beta_{\perp}$ is that the relative magnetic dipole strength of the reflecting medium to the incident medium is greater than the similar relative electric dipole strength, plus the additional condition that the product of the electric and magnetic dipole strengths of the reflecting medium is greater than the similar product for the incident medium. Replace greater by lesser in the preceding argument and we have the other existence condition for $\beta_\perp$. Contrast this to a similar existence condition for $\beta_\parallel$ where the relative magnetic dipole strength of the incident medium is also greater than the relative electric dipole strengths but the dipole strength product of the reflecting medium is less than that of the incident medium. In summary, the material properties that determine the existence of a Brewster angle for an incident

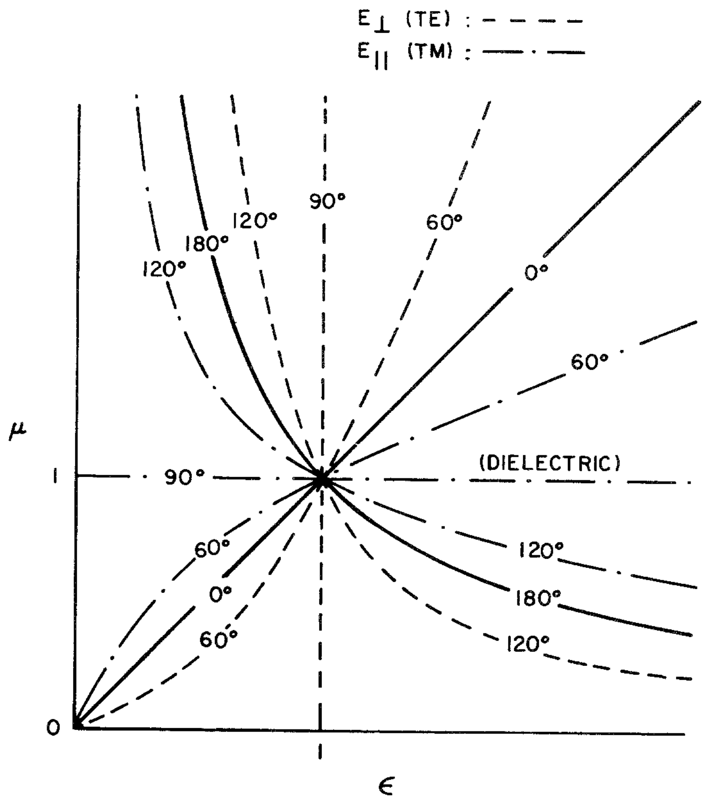

Fig. 4. Curves of constant $\alpha \equiv \beta + \theta_2$ are illustrated as a function of ε and μ. When either $\varepsilon = 1$ or $\mu = 1$, we have the well-known condition $\alpha = 90°$. If $\varepsilon = \mu$ (impedance matching), $\alpha = \beta = 0°$ (Brewster's angle at normal incidence); and if $\varepsilon = 1/\mu$ (refractive index matching), $\alpha = 180°$ (Brewster's angle at grazing incidence).

polarization are more complex that might be expected. But simple inequalities between the relative permittivity and the relative permeability or between the relative refractive index and the relative impedance provide definitive existence conditions.

It is also interesting to consider how the permeability changes the orthogonality condition between the Brewster angle and transmitted beam angle, i.e., how does $\beta+\theta_2$ behave (see figure 1) for general ε and μ? Referring to equations (3) and (4), we easily see that

$$\text{polarization } E_\perp : \qquad \cos(\beta+\theta_2) = \sqrt{(\mu/\varepsilon)}\ \frac{1-\varepsilon}{1-\mu}$$

$$\text{polarization } E_\parallel : \qquad \cos(\beta+\theta_2) = \sqrt{(\varepsilon/\mu)}\ \frac{1-\mu}{1-\varepsilon} \quad . \qquad (10)$$

Figure 4 illustrates a family of curves of $\alpha \equiv \beta+\theta_2$ for variable ε,μ. We see that $\alpha = 90°$ only when $\varepsilon = 1$ or $\mu = 1$. These values of α will vary from 0° to 180°, depending on ε and μ. For the case where $\varepsilon = \mu$ we have the Brewster angle at normal incidence.[4] For the case where $\varepsilon = 1/\mu$ (discussed at length in reference 1), the Brewster angle is at extreme grazing incidence; this may be a singular situation if $\varepsilon_1\mu_1 < \varepsilon_2\mu_2$ (or $\varepsilon < 1/\mu$) since a critical angle also exists--the Brewster angle always precedes the critical angle but if $\varepsilon = 1/\mu$ each "coexists" at $\theta_1 = \beta = 90°$. All curves pass through the point $\varepsilon = \mu = 1$; this is not a pathological situation, rather, no electromagnetic interface exists for these material parameters. Realizing that magnetic dipoles will radiate the same as electric dipoles, the contribution of each (parameterized by μ and ε) will lead to the results shown in figure 4. We should also point out that for a nonlinear optical material whereby ε and/or μ can be electrically (or thermally) modulated, a form of millimeter wave optical "switch" can be devised. This, however, can only be realized if suitable materials can be located or fabricated.

We believe that this work will interest millimeter wave researchers concerned with the development and characterization of magnetic materials. Further applications of this and previous studies include interesting scattering properties of magnetic aerosols for millimeter and FIR lasers, and the application to Brewster angle measurements for the determination of the optical properties of magnetic media (similar to that for dielectrics in the visible).[2,8]

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